Math CAMPPP 2011

Plenary 4

Responding to Student Work Over Time

Students Responding to Students

Assessing Algebraic Reasoning

Some Assumptions

- Mathematical knowledge is constructed in the course of engaging in interrelated activities
- Constructing mathematical knowledge develops through students' making connections among their intuitions, theories, experiences, and previously disconnected fragments of knowledge

Some More Assumptions

- Learning is non-linear – sophisticated strategies often used with less-sophisticated
- Students can leap from recursive to explicit reasoning
- Students can leap from visual to numeric to symbolic representations
- This researched method of working with linear growing patterns is an intervention that supports ALL students

Revealing Student Thinking

- In order to respond to student work, we have to create opportunities for students to demonstrate their thinking
- We need use tasks that encourage students to describe, explain, represent, and justify their thinking
- We need to NOTICE student thinking (Mason)
- In turn, students will be able to judge for themselves whether their responses "make sense"

Evaluating Student Thinking About Linear Relationships

- What kind of Generalizing strategy does the student use?
  - (guess and check, counting, recursion, explicit rule) – sometimes more than one of these strategies is evident, particularly as students move from near to far predictions, and to providing a general rule

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Evaluating Student Thinking About Linear Relationships

- Evidence of Justification
  - Does the student provide evidence about why their rule is correct?

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Evaluating Student Thinking About Linear Relationships

Can the student:

- Show multiple solutions that are reasonable?
- Illustrate his/her ideas using more than one effective representation?
- Move amongst different representations to justify thinking?
- Express relationships using accurate syncopated notation (i.e., words, numbers, letters and/or symbols) or more formal algebraic notation?

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Evaluating Three Work Samples

- Pine Tree
- Analyse the samples (based on prior knowledge and what you've learned at CAMPPP) – use STICKIES to label the student work with your comments
- What would you write to the student?
- Has the student demonstrated their understanding?
- What does the student need now?

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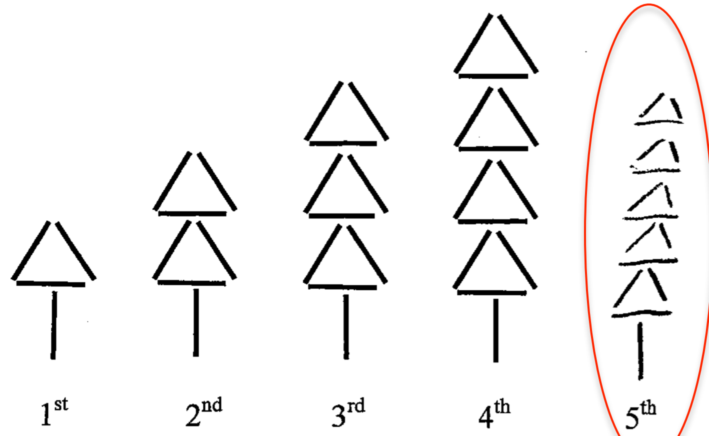
Student A

Drawing & Counting for P 20

PineTrees

These are pictures of pine trees made out of toothpicks.



Checks with pictorial representation

If this pattern continues to increase with the addition of one more triangle each time, how many toothpicks would you need to make the next (5th) tree? 16.

How do you know that? Because P1 has 4 P2 has 7 and P3 has 10. It just keeps going up by 3 each time.

Additive Thinking

How many toothpicks would you need to make the 9th tree? 28

How do you know that? Because 4 more times 3 on 16 is 28.

Multiplicative / Chunking

How many toothpicks would you need to make the 100th tree? 301

How many toothpicks would you need to make the 43rd tree?

What's the rule?

X3 each time + 1

Explicit Rule (is it used?)

Bonus Question

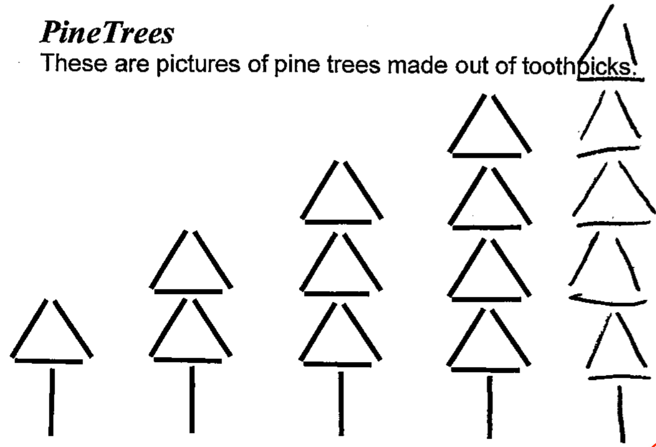
If you had 61 toothpicks could you make a tree that fits this pattern? What position would it be in? P 20.

Accurate

Student B

PineTrees

These are pictures of pine trees made out of toothpicks.



1st 2nd 3rd 4th 5th

6 19 7 22 8 25 9 28

> Finding the total without drawing but increasing sequentially

If this pattern continues to increase with the addition of one more triangle each time, how many toothpicks would you need to make the next (5th) tree?

How do you know that? You would need sixteen toothpicks to make the fifth tree.

How many toothpicks would you need to make the 9th tree?

How do you know that? You would need 28 toothpicks to make the 9th. I just added 3 toothpicks each time.

> Additive Thinking

150 − 35 = 115
−35

How many toothpicks would you need to make the 100th tree? 301

How many toothpicks would you need to make the 43rd tree? 115

What's the rule? PN X3+1.

> Explicit Rule, But is it used?

> One is accurate

Bonus Question

If you had 61 toothpicks could you make a tree that fits this pattern? What position would it be in? No because the last number is always 9, 2, 5, 8. No the closest you could get would be 60.

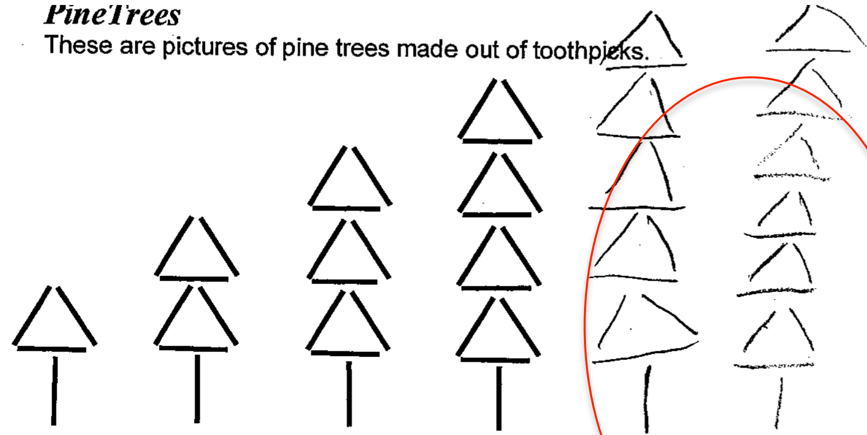
26 × 3 = 78 23 × 3 = 69 20 × 3 = 60

> Loses track of the constant

Student C

PineTrees

These are pictures of pine trees made out of toothpicks.



1^{st} 2^{nd} 3^{rd} 4^{th} 5^{th} 9 th

Draws 5^{th} and 6^{th} Terms

If this pattern continues to increase with the addition of one more triangle each time, how many toothpicks would you need to make the next (5^{th}) tree? How do you know that?

5x3=15+1=16

Accurately calculates

How many toothpicks would you need to make the 9^{th} tree? How do you know that?

9x3=27+1=28

How many toothpicks would you need to make the 100^{th} tree? 100x3=300+1=30

How many toothpicks would you need to make the 43^{rd} tree? 43x3=127

What's the rule?

____ x 3 = ____ + 1 = ____

figure # answer answer

Generalized rule notation follows calculation notation

Bonus Question

If you had 61 toothpicks could you make a tree that fits this pattern? What position would it be in?

20x3=60 +1=61

60÷3=20

It could make figure #20

Uses same calculation but why 20? *Then* shows division.

Evaluating Student Thinking About Linear Relationships

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Evaluating Student Thinking About Linear Relationships

- Evidence of Justification
  - Does the student provide evidence about why their rule is correct?

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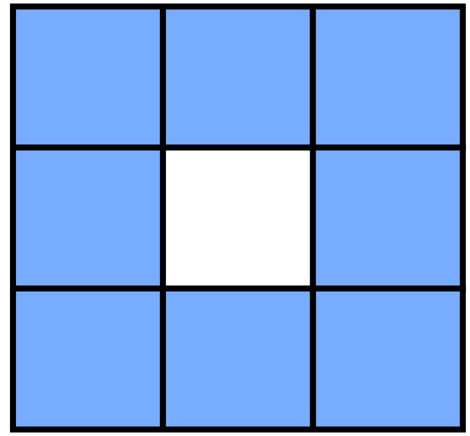
Demonstrating an Understanding of Linear Growing Patterns

Can these students:

- Show multiple solutions that are reasonable?
- Illustrate his/her ideas using more than one effective representation?
- Move amongst different representations?
- Express relationships using accurate syncopated notation (i.e., words, numbers, letters and/or symbols) or more formal algebraic notation?

Border Problem



- This is a 3x3 grid of squares with only the outside edge shaded.
- If you had a 17x17 grid of squares with only the outside edge of squares shaded, how many squares would be shaded?
- What is your rule for finding the shaded squares for any size of grid?

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What Solution(s) Did You Find?

- Pairs to Squares or Table Talk
- Explain and show your thinking
- Ask one another Key Questions that press for justification

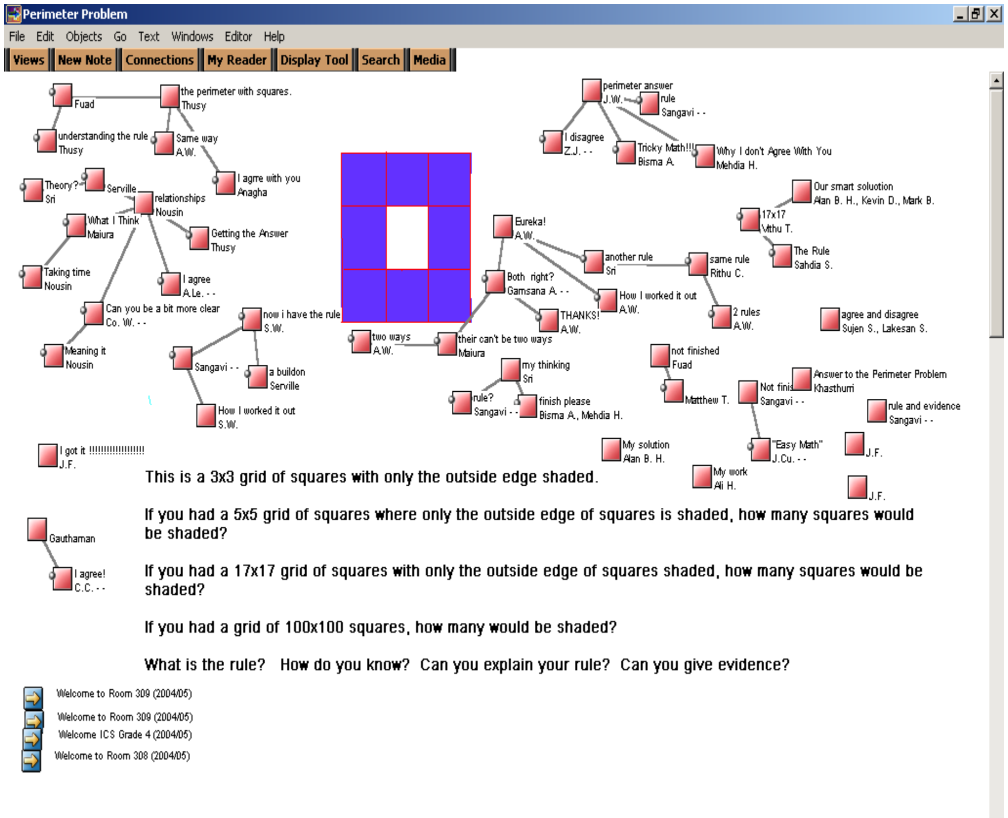
Students Responding To Students

- Assessment As Learning includes self and peer assessment
- How does students' assessment of one another's mathematical thinking enhance their own thinking? How does it enhance the mathematical thinking of others?
- Research of Grade 6 students who had never met, corresponded about their solutions to generalizing problems in an asynchronous discourse space

Knowledge Forum

Bereiter and Scardamalia

- Knowledge Forum is a networked multimedia knowledge space created by community members.
- Knowledge building is supported through co-authored notes, and the ability to build on to ongoing discussions.
- Provides authentic situation for mathematizing
- Students respond to and question one another



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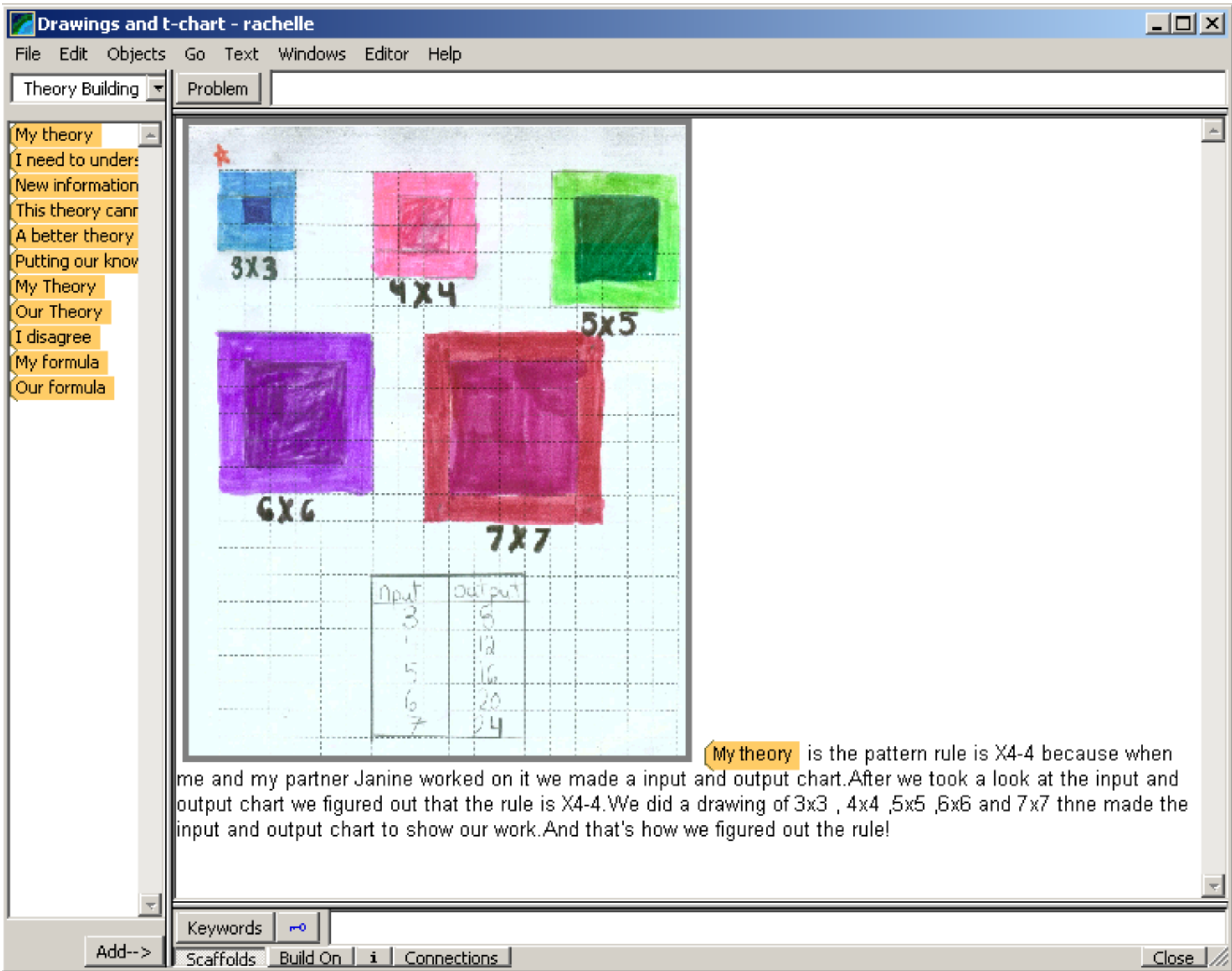
Sample Grade 4 Response



Asking For Justifications

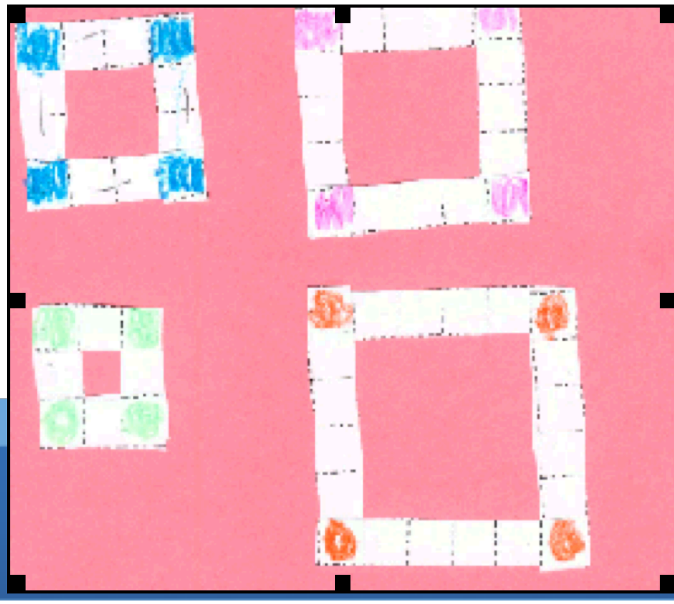
Our theory – JN, KB, JN (Classroom 1)

is that the rule is x4-4

Why? – KV (Classroom 2) Why is that your rule?

Explaining our theory – JN,KB, JN (Classroom 1)

because when the corners overlap it takes away 4 and it is times 4 because there is 4 sides on each square.WE EXPLAINED OUR THEORY...YaY!!!...



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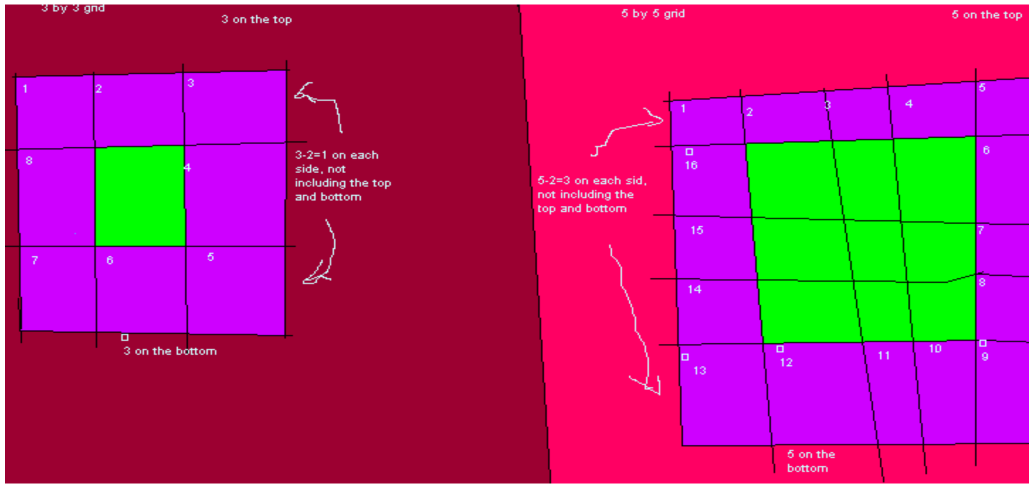
Identifying Multiple Solutions



Perimeter Rule CT (Classroom 2) - The slow way of getting to the answer is to draw the grid, as I've done above. The fast way is calculations in your head, or you can do the medium fast way, calculating on paper. Or the illegal calculator. The calculations are simple. If it's a 9 by 9 grid, you calculate 9+9. Then 9-2x2. Then add the results. In this case it's 18+14. Which equals to 36.**If you translated that into a rule the rule would be, #+#+(#-2x2).**

Discussing Multiple Solutions

Eureka! AW (Lab)

for the 5x5 question you do 5x5=25 the square of 25 is 5 and you minus two from the square and square that then minus it from your original number and you have your answer!

First I drew the five by five grid and there was nine in the middle to take away - 3x3=9

So then I figured out a 6x6 square was 36 and I know that inside there would be a 4x4 square to take away so the difference between 6 and 4 is 2 - so it was 36-16=20

Then 17x17=289 the square root is 17 then minus 2 from 17 which is 15 (because before there was 2 difference like between 6 and 4) and then 15x15=225 then I minused 225 from 289 and got 64

nxn=nsquared -(n-2)squared

Another rule – SI (Pub)
I have another rule for you and it is the input x4-4. In the rule it is x4 because there is 4 sides in a square. It is -4 because when you multiply 4 you are repeating the corners twice so you –4.

2 rules – AW (Lab)
but there might be two rules because we got the same answer for both so I think there is more then 1 way to figure the problem out

Both right? – GA (Pub)
I agree with you and disagree with you because you've got the answer but in a complicated way. I disagree with you because there's an easier way than taking the square of 25, subtracting 2 from it and square that and then subtract that from your original answer. I agree with you because for the first few questions you got it right.

Two Ways – AW (Lab)
Why can't there be two ways. There are different ways to do lots of different problems I think you can have two ways nxn=n squared -(n-2) squared works and x4-4 works

Assessment As Learning

- Students negotiated multiple “seeings” resulting in multiple solution strategies for the problem
- Students created an expectation for providing justifications for their solutions, not just the “pattern rules”
- Over time, students demonstrated an increasing commitment and sophistication of justifications

Assessment As Learning

- Students established a community norm of routinely offering notes that included explanations of their answers using language, tables of values, symbols and/or images to justify their conjectures
- This commitment to explain their thinking to others deepened their own mathematical thinking, and helped them to critique the thinking of others by questioning conjectures, and offering alternative solutions

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